Cambridge Assessment Admissions Testing

Test of Mathematics for University Admission

Paper 1 practice paper hand-written worked answers

Admissions Testing Service

TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

D513/11

Model Answers

PAPER 1

Morning

Practice paper

Time: 75 minutes

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Additional Materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only points for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators and dictionaries must **NOT** be used.
There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

This question paper consists of 13 printed pages and 3 blank pages.

PV2

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1 It is given that the expansion of $(ax+b)^3$ is $8x^3 - px^2 + 18x - 3\sqrt{3}$, where a, b and p are real constants.

$(ax+b)^3 = (ax+b)(a^2x^2 + 2abx + b^2)$

What is the value of p?

$= a^3x^3 + 2a^2bx + ab^2x + a^2bx^2 + 2ab^2x + b^3$

$= a^3x^3 + (3a^2b)x^2 + (3ab^2)x + b^3$

A $-12\sqrt{3}$

B $-6\sqrt{3}$ $a^3 = 8 \Rightarrow a = 2$

C $-4\sqrt{3}$ $3ab^2 = 18$ $b^3 = -3\sqrt{3}$

D $-\sqrt{3}$ $6b^2 = 18$ $b = -\sqrt{3}$

E $\sqrt{3}$ $b^2 = 3$

F $4\sqrt{3}$ $b = \pm\sqrt{3}$

G $6\sqrt{3}$

$\boxed{\text{H } 12\sqrt{3}}$ $3a^2b = -p$

$-p = 3 \times 4 \times -\sqrt{3}$

$-p = -12\sqrt{3}$

$p = 12\sqrt{3}$

2 The expression $3x^3 + 13x^2 + 8x + a$, where a is a constant, has $(x+2)$ as a factor.

Which one of the following is a complete factorisation of the expression?

$f(x) = 3x^3 + 13x^2 + 8x + a$

A $(x+2)(x-1)(3x-2)$ $f(-2) = 0 = 3 \times -8 + 13 \times 4 + 8 \times -2 + a = 12 + a$

B $(x+2)(x+1)(3x-2)$

C $(x+2)(x+1)(3x+2)$ $\therefore 12 + a = 0 \Rightarrow a = -12$

D $(x+2)(x-3)(3x+2)$ $f(x) = (x+2)(3x^2 + Bx + C)$

$\boxed{\text{E } (x+2)(x+3)(3x-2)}$ $= 3x^3 + Bx^2 + Cx + 6x^2 + 2Bx + 2C$

F $(x+2)(x+3)(3x+2)$ $= 3x^3 + (B+6)x^2 + (C+2B)x + 2C$

$B + 6 = 13$ $2C = -12$

$B = 7$ $C = -6$ $\therefore f(x) = (x+2)(3x^2 + 7x - 6)$

$= (x+2)(x+3)(3x-2)$

3 A line is drawn normal to the curve $y = \frac{2}{x^2}$ at the point on the curve where $x = 1$. $(1, 2)$

Gradient of curve $= dy/dx = -4x^{-3}$

This line cuts the x-axis at P and the y-axis at Q. Gradient at $(1,2)$ is -4

The length of PQ is Gradient of normal at $(1,2)$ is $1/4$

A $\frac{3\sqrt{5}}{2}$

B $\frac{3\sqrt{17}}{4}$

C $\frac{7\sqrt{17}}{4}$

D $\frac{35}{4}$

E $\frac{35\sqrt{5}}{2}$

F $\frac{3\sqrt{17}}{2}$

Equation of normal is of form $y = mx + c$

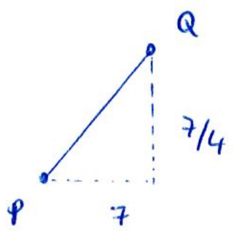
$2 = \frac{1}{4} \times 1 + c$

$c = 7/4$

$y = \frac{1}{4}x + \frac{7}{4}$

when $x = 0$, $y = 7/4 \longrightarrow Q\ (0, 7/4)$

$y = 0$, $x = -7 \longrightarrow P\ (-7, 0)$



$(PQ)^2 = 7^2 + (7/4)^2 = 49 + 49/16 = 833/16$

$\therefore PQ = \frac{7\sqrt{17}}{4}$

4 The sequence a_n is defined by the rule:

$$a_n = (-1)^n - (-1)^{n-1} + (-1)^{n+2} \text{ for } n \geq 1.$$

Find the value of

$$\sum_{n=1}^{39} a_n$$

A -39

B -3

C -1

D 0

E 1

F 3

G 39

$a_1 = (-1)^1 - (-1)^0 + (-1)^3 = -1 - 1 - 1 = -3$

$a_2 = (-1)^2 + (-1)^1 + (-1)^4 = 1 - -1 + 1 = 1 + 1 + 1 = 3$

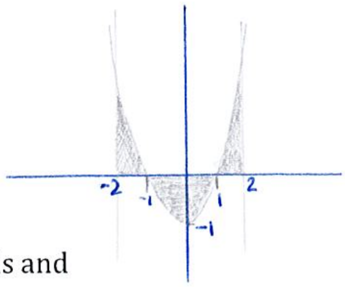
$a_3 = (-1)^3 - (-1)^2 + (-1)^5 = -1 - 1 - 1 = -3$

Sequence is $\underbrace{-3, 3}_{0}, \underbrace{-3, 3}_{0}, \dots, -3$

$\sum_{n=1}^{39} a_n = -3$

$x = -2 \Rightarrow y = 3$ points $(-2, 3)$

$x = 2 \Rightarrow y = 3$ $(2, 3)$



5 What is the total area enclosed between the curve $y = x^2 - 1$, the x-axis and the lines $x = -2$ and $x = 2$?

A $\frac{4}{3}$

B $\frac{8}{3}$

$\boxed{\text{C} \quad 4}$

D $\frac{16}{3}$

E 12

F 16

$$\int_1^2 x^2 - 1\,dx = \int_{-2}^{-1} x^2 - 1\,dx = \frac{x^3}{3} - x \Big|_1^2 = \left(\frac{2^3}{3} - 2\right) - \left(\frac{1}{3} - 1\right)$$

$$= \frac{8}{3} - 2 - \frac{1}{3} + 1 = \frac{4}{3}$$

$$\int_{-1}^{1} x^2 - 1\,dx = \left(\frac{1}{3} - 1\right) - \left(\frac{-1}{3} + 1\right) = \frac{1}{3} - 1 + \frac{1}{3} - 1 = \frac{-4}{3}$$

$$\text{Area} = \frac{4}{3} + \frac{4}{3} + \frac{4}{3} = 4$$

6 P, Q, and R are each mixtures of red and white paint.

The percentage by volume of red paint in P is 30%.

The percentage by volume of red paint in Q is 20%.

The mixtures P, Q, and R are combined in the proportion 12 : 5 : 3 respectively.

If the resulting mixture contains 25% by volume of red paint, what percentage by volume of mixture R is red paint?

A 25%

B 23%

$\boxed{\text{C} \quad 13\tfrac{1}{3}\%}$

D $19\frac{1}{2}$%

E $9\frac{3}{4}$%

F It is impossible to achieve this result.

paint	% Red	Volume	volume Red
P	30	12	$12 \times 30/100 = 3.6$
Q	20	5	$5 \times 20/100 = 1$
R	R	3	$3 \times R/100 = 3R/100$
Total	25	20	$20 \times 25/100 = 5$

So: $3.6 + 1 + \frac{3R}{100} = 5$

$$\frac{3R}{100} = \frac{2}{5}$$

$$R = \frac{40}{3} = 13\tfrac{1}{3}$$

Assume 300 members to make this easier (300 is divisible by 3, 5 & 100)

[redacted]